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Comment on "Lateral Vibration and Stability Relationship of Elastically Restrained Circular Plates"

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RAO and Amba-Rao[1] have presented the results of Rayleigh-Ritz calculations for the vibration frequencies of radially-compressed and elastically restrained circular plates and have shown that a plot of the buckling load vs a frequency parameter, λ, which they describe as the "natural frequency," is nonlinear. In fact, their λ is proportional to the square root of the natural frequency according to the definition given in their nomenclature, as may also be readily verified from the known solutions for the frequencies of circular plates without radial loading.[2] Moreover, there is no reason to expect the relationship between either the natural frequency or the square root of the natural frequency and the radial load to be linear, since the usual linear relationship is between the square of the frequency and the load.

The well-known relationship between compressive load and frequency squared which applies for the case of a column (and also for the critical speed of a compressed shaft) follows from Southwell's theorem[3] in the form of a lower bound

$$\omega^2/\omega_0^{\,2} + P/P_0 \geqq 1$$

where P is the column load, P_0 is the column buckling load, ω is the lowest natural frequency and ω_0 is the lowest natural frequency in the absence of load P. The proof of this inequality is straightforward and is based on Rayleigh's principle.

For pin-ended columns, the deflection curves in vibration and in buckling are identical, and the equal sign applies. However, in a great many other cases, this lower-bound inequality is also very nearly an equality. Southwell's theorem also applies to the case of additional loadings which may in themselves cause instability. Care is needed in choosing the frequency and loading parameters in the governing differential equations or energy functionals so that Southwell's theorem will apply. In the case of the radially-compressed circular plate, it is clear from the energy functionals[2,4] that the linear inequality is between frequency squared and compressive load.

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It turns out in this case, as in many others, that the Southwell inequality is extremely close to an equality so that a straight line is an excellent approximation to the curve of frequency-squared vs compressive load. (It is also worth noting that the inequality also applies to tensile load if the sign of P is reversed.) The near equality of the Southwell inequality may be readily demonstrated by calculating for the clamped plate the function ϕ defined by

$$\phi = R/R^* + (\lambda/\lambda^*)^4 \tag{1}$$

where the notation is that of Ref. 1, in which R is the compressive load, R^* is the buckling load, λ is the frequency parameter referred to above, and λ^* is the value of the frequency parameter in the absence of compressive load. This gives, using the data in Ref. 1, for R and λ

Table 1 Frequency-load relationship

R/R^*	$(\lambda/\lambda^*)^4$	ϕ
0	1.0	1.0000
0.1361	0.8690	1.0051
0.2721	0.7313	1.0034
0.4082	0.6030	1.0102
0.5442	0.4551	0.9993
0.6827	0.3258	1.0085
0.8163	0.1807	0.9970
0.9524	0.0404	0.9928
1.0	0	1.0000

More decimal places have been retained here than are warranted by the accuracy of the data in Ref. 1, so that the deviations of ϕ from unity are undoubtedly of the order of the round-off errors involved. Thus, to a high degree of approximation

$$R/R^* + (\lambda/\lambda^*)^4 = R/R^* + \omega^2/\omega_0^{\,2} \approx 1.0 \tag{2}$$

and Southwell's theorem in this case may be regarded as an equality to an accuracy far greater than that required for all practical engineering purposes.

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WE thank Mr. Flax for his interesting comments. The present work concerning the vibration and stability relationship of structural elements is a part of a continuing program. Regretfully,

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